HW to Chapter 3 & 4 “The Perceptron for Logistic Regression

Non-programming Assignment:

1. Describe the logistic regression

Logistic regression is a process of modeling the probability of a discrete outcome given an input variable. It is typically used for binary classification problems where the goal is to determine if an instance belongs to one of two categories. The logistic regression model uses a sigmoid activation function to predict the probability that the output belongs to a predefined class (for example, a "yes/no" or "true/false" classification). The output is a probability value between 0 and 1, which is then classified into one of the categories based on a threshold (usually 0.5).

2. How are grayscale and color (RGB) images presented as inputs for the perceptron?

Grayscale Images: A grayscale image is represented by pixel intensities between 0 and 255. If an image is 20×2020 \times 2020×20 pixels, it is mapped into a vector of size 400, with each element corresponding to the pixel value. RGB Images: In the case of color images, each pixel contains three values representing the intensities of the red, green, and blue channels. A 20×2020 \times 2020×20-pixel RGB image would be represented as a vector of size 3×20×20=12003 \times 20 \times 20 = 12003×20×20=1200, where each set of 400 values represents the intensities of one-color channel (Red, Green, or Blue)

3. Is image recognition a logistic regression problem? Why?

Yes, image recognition can be framed as a logistic regression problem. Logistic regression can be used to classify images into predefined categories, such as detecting whether an image contains a cat or not. The features (like pixel intensities) of the image are used as inputs, and the output is a probability that the image belongs to a certain class. Logistic regression is commonly applied for binary classification tasks, which fits the nature of image recognition if it’s set up as a binary classification problem (e.g., "cat" or "not cat")

4. Is home prices prediction a logistic regression problem? Why?

No, home price prediction is not typically a logistic regression problem. Logistic regression is used for classification tasks, while predicting home prices is a regression problem. The goal of regression is to predict continuous values (e.g., the price of a house), not binary or categorical outcomes. In this case, linear regression or other forms of regression would be more appropriate for predicting house prices based on features like size, number of rooms, etc.

5. Is image diagnostics a logistic regression problem? Why?

Yes, image diagnostics can be framed as a logistic regression problem if the task is to classify the image into categories (e.g., healthy vs. unhealthy). In medical diagnostics, for example, logistic regression can be used to predict the probability of a disease based on input features derived from the image and classify it accordingly. Logistic regression works well for such binary classification problems, which makes it suitable for tasks like image diagnostics where the output is a probability of a diagnosis.

6. How does gradient descent optimization work?

Gradient descent is an optimization algorithm used to minimize the cost function by iteratively adjusting the model parameters (such as weights and biases). In each step of gradient descent: The model computes the prediction based on the current weights and biases. It calculates the cost (or loss) using a loss function (e.g., cross-entropy for logistic regression). It computes the gradient of the cost with respect to the parameters, which indicates the direction of the steepest ascent. The weights and biases are updated in the opposite direction of the gradient (descent) to reduce the cost. This process continues until the model converges to a minimum of the cost function.

7. How does image recognition work as logistic regression classifier?

In image recognition using logistic regression, the pixel values of an image (either grayscale or RGB) are treated as input features. Logistic regression computes a weighted sum of these input features and passes the result through a sigmoid activation function. The sigmoid function outputs a probability that the image belongs to a certain class. Based on this probability, the image is classified into one of two categories (e.g., "cat" or "not cat"). Logistic regression works for binary classifications, but for more complex image recognition tasks, more advanced models like convolutional neural networks (CNNs) are often used.

8. Describe the logistic regression loss function and explain the reasons behind this choice.

The logistic regression loss function is called the cross-entropy loss or log loss. For a single training example, it is expressed as:

J(a,y)=−(ylog⁡(a)+(1−y)log⁡(1−a))J(a, y) = -(y \log(a) + (1 - y) \log(1 - a))J(a,y)=−(ylog(a)+(1−y)log(1−a))

Where a is the predicted probability (output of the sigmoid function), y is the actual label (0 or 1). This loss function penalizes large differences between the predicted probability and the actual label. It is convex, which ensures that gradient descent will converge to a global minimum. The cross-entropy loss function is preferred in binary classification tasks because it provides a clear way to measure the divergence between the predicted probabilities and the actual labels, which in turn guides the learning process during training.

9. Describe the sigmoid activation function and the reasons behind its choice.

The sigmoid activation function is defined as:

σ(z)=11+e−z\sigma(z) = \frac{1}{1 + e^{-z}}σ(z)=1+e−z1

Where zzz is the weighted sum of inputs to the neuron. The sigmoid function is chosen for logistic regression because: It outputs a value between 0 and 1, which can be interpreted as a probability. It is differentiable, which allows for the use of gradient descent during the learning process. It has a smooth gradient, which prevents abrupt changes in the weight updates and ensures smoother learning. The sigmoid function helps in converting the linear combination of inputs into a non-linear output, which is crucial for binary classification problems like logistic regression.